

(c) Show that :

$$A = \begin{bmatrix} i & 0 & 0 \\ 0 & 0 & i \\ 0 & i & 0 \end{bmatrix}$$

is Skew-Hermitian and also unitary.

Printed Pages : 8

EAS-103/ASM-101

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9601

Roll No.

1	1	1	4	3	1	0	0	9	6
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B. Tech.

(Semester-I) Theory Examination, 2011-12

MATHEMATICS-I/ENGINEERING MATHEMATICS-I

Time : 3 Hours]

{Total Marks : 100

*Note : Attempt questions from each Section as indicated.
The symbols have their usual meaning.*

Section-A

Attempt *all* parts of this question. Each part carries 2 marks. 2×10=20

1. (a) Find y_n if $y = \frac{x^n - 1}{x - 1}$.
- (b) What is the asymptote of the curve $y^2(2a-x) = x^3$?
- (c) If $u = x^2yz - 4y^2z^2 + 2xz^3$, find the value of :

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z}$$

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(d) Calculate:

$$\frac{\partial(u, v)}{\partial(x, y)}$$

for $x = e^u \cos v$ and $y = e^u \sin v$.

(e) Find the value of:

$$\sqrt{\left(-\frac{5}{2}\right)}$$

(f) Find the value of the integral $\iint_R xy \, dx \, dy$, where R is the region bounded by the x -axis, the line $y = 2x$ and the parabola $x^2 = 4ay$.

(g) Show that the vector:

$$\vec{V} = 3y^4 z^2 \hat{i} + 4x^3 z^2 \hat{j} - 3x^2 y^2 \hat{k}$$

is solenoidal.

(h) State Green's theorem for a plane region.

(i) The matrix:

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

satisfies the matrix equation $A^3 - 6A^2 + 11A - I = 0$, where I is an identity matrix of order 3. Find A^{-1} .

(j) Show that the matrix:

$$A = \begin{bmatrix} 3 & 1 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$$

is diagonalizable.

Section-B

Attempt any *three* parts of this question. 10×3=30

(a) If $y(x) = \sin px + \cos px$, prove that:

$$y_n(x) = p^n [1 + (-1)^n \sin 2px]^{1/2}.$$

Hence, show that $y_8(\pi) = \left(\frac{1}{2}\right)^{31/2}$ when $p = \frac{1}{4}$.

(b) Evaluate:

$$[(3.82)^2 + 2(2.1)^3]^{1/5}$$

using theory of approximation.

(c) Evaluate:

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \frac{e^y}{(e^y + 1)\sqrt{1-x^2-y^2}} \, dx \, dy.$$

(d) Evaluate:

$$\iint_S \vec{F} \cdot \hat{n} \, dS,$$

where $\vec{F} = 18z\hat{i} - 12\hat{j} + 3y\hat{k}$ and S is the part of the plane $2x+3y+6z=12$ in the first octant.

(e) If:

$$A = \begin{bmatrix} 1 & 2 & 1 \\ a & 0 & 4 \\ 1 & 1 & 1 \end{bmatrix}$$

and $\text{adj}(\text{adj} A) = A$, find a .

Section-C

All questions of this Section are compulsory. Attempt any two parts from each question. $10 \times 5 = 50$

3. (a) If $y = \left(\frac{1+x}{1-x}\right)^{1/2}$, prove that:

$$(1-x^2)y_n - [2(n-1)x+1]y_{n-1} - (n-1)(n-2)y_{n-2} = 0.$$

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(4)

(b) Obtain the series for $\log_e(1+x)$ and then find the series for $\log_e\left(\frac{1+x}{1-x}\right)$ and hence, determine the value of $\log_e\left(\frac{11}{9}\right)$ up to five places of decimal.

(c) Find the asymptotes of:

$$y^3 - x^2y - 2xy^2 + 2x^3 - 7xy + 3y^2 + 2x^2 + 2x + 2y + 1 = 0.$$

4. (a) If:

$$u = \sin^{-1}\left(\frac{x^{1/3} + y^{1/3}}{x^{1/2} - y^{1/2}}\right)^{1/2},$$

show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = -\frac{1}{12} \tan u$.

(b) Are the functions:

$$u = \frac{x-y}{x+z}, \quad v = \frac{x+z}{y+z}$$

functionally dependent? If so, find the relation between them.

(c) Using the Lagrange's method, find the maximum and minimum distances from the origin to the curve $3x^2 + 4xy + 6y^2 = 140$.

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(5)

5. (a) Prove that :

$$B(m, n) = \int_0^1 \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}} dx.$$

- (b) Find the volume of the tetrahedron bounded by the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ and the coordinate planes.

- (c) Find the volume of the solid which is bounded by the surfaces $2z = x^2 + y^2$ and $z = x$.

6. (a) If $\vec{F} = (\vec{a} \cdot \vec{r}) \vec{r}$, where \vec{a} is a constant vector, find $\text{curl } \vec{F}$ and prove that it is perpendicular to \vec{a} .

- (b) Find the work done in moving a particle in the force field :

$$\vec{F} = 3x^2\hat{i} + (2xz - y)\hat{j} + z\hat{k}$$

along the curve $x^2 = 4y$ and $3x^3 = 8z$ from $x = 0$ to $x = 2$.

- (c) Prove that :

$$\iint_S \frac{1}{\sqrt{a^2x^2 + b^2y^2 + c^2z^2}} dS = \frac{4\pi}{\sqrt{abc}},$$

where S is the ellipsoid $ax^2 + by^2 + cz^2 = 1$.

7. (a) Find the value of P for which the matrix :

$$A = \begin{bmatrix} 3 & P & P \\ P & 3 & P \\ P & P & 3 \end{bmatrix}$$

is be of rank 1.

- (b) Show that the system of equations :

$$3x + 4y + 5z = A$$

$$4x + 5y + 6z = B$$

$$5x + 6y + 7z = C$$

are consistent only if A, B and C are in arithmetic progression (A. P.).