(f) Apply Dirichlet's integral to find the volume of the solid bounded by the co-ordinate plane and the surface

$$\sqrt{\frac{x}{a}}+\sqrt{\frac{y}{b}}+\sqrt{\frac{z}{c}}=1.$$

5. Attempt any **TWO** parts of the following : **(10×2=20)**

(a) (i) If $\vec{F}(x,y,z)=xz^3\hat{i}-2x^2yz\hat{j}+2yz^4\hat{k}$, find curl of

(ii) Find the directional derivative of in the direction of $\vec{r}$, where

(b) Find the circulation of round the curve C, where and C is the rectangle whose vertices are (0, 0), (1, 0), (1, π/2) and

(c) State the Green's theorem. Apply this theorem to evaluate :

$$\int_C (x^2+xy)dx+(x^2+y^2)dy,$$

where C is the square formed by the lines y = ± 1, x = ± 1.

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9916 **Roll No.** | | | | | | | | | | |

B.Tech.

(SEM. I) ODD SEMESTER THEORY EXAMINATION 2012-13

MATHEMATICS—I

Time : 3 Hours *Total Marks : 100*

Note :—*(1) Attempt all questions.*

(2) All questions carry equal marks.

(3) The symbols have their usual meaning.

1. Attempt any **TWO** parts of the following : **(10×2=20)**

(a) Find the rank of the following matrix by reducing it to normal form :

[illegible] $e^x \cos y$

$$\begin{bmatrix} 1 & 2 & -1 & 3 \\ 4 & 1 & 2 & 1 \\ 3 & -1 & 1 & 2 \\ 1 & 2 & 0 & 1 \end{bmatrix}.$$

(b) Investigate, for what values of λ and μ do the system of equations

$$x + y + z = 6,$$
$$x + 2y + 3z = 10,$$
$$x + 2y + \lambda z = \mu$$

have :

(i) no solution

(ii) unique solution

(iii) infinite solutions ?

(c) Verify Cayley-Hamilton theorem for the matrix

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}.$$

Hence compute A^{-1}.

2. Attempt any **FOUR** parts of the following : **(5×4=20)**

(a) If $y = (\sin^{-1} x)^2$, prove that

$$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2 y_n = 0.$$

(b) If $z = x^2 \tan^{-1}\left(\frac{y}{x}\right) - y^2 \tan^{-1}\left(\frac{x}{y}\right)$, prove that

$$\frac{\partial^2 z}{\partial y \partial x} = \frac{x^2 - y^2}{x^2 + y^2}.$$

(c) If $u = \log_e\left(\frac{x^4 + y^4}{x + y}\right)$, show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3.$$

(d) Trace the curve :

$$y^2(a + x) = x^2(3a - x).$$

(e) If $V = f(x^2 + 2yz, y^2 + 2xz)$, find the value of

$$(y^2 - xz)\frac{\partial v}{\partial x} + (x \ldots)\frac{\partial v}{\partial y} + (z \ldots xy)\frac{\partial y}{\partial z}$$

(f) Expand $x^2y + 3y - 2$ in powers of $(x - 1)$ and $(y + 2)$ using Taylor's theorem.

3. Attempt any **TWO** parts of the following : **(10×2=20)**

(a) (i) Verify the chain rule for Jacobians, if

$$x = u, \quad y = u \tan v, \quad z = w.$$

(ii) If $u = x + 2y + z$, $v = x - 2y + 3z$ and $w = 2xy - xz + 4yz - 2z^2$, show that they are not independent. Find the relation between u, v and w.

(b) If Δ is the area of a triangle, prove that the error in Δ resulting from a small error in C is given by

$$\delta\Delta = \frac{\Delta}{4}\left[\frac{1}{s} + \frac{1}{s-a} + \frac{1}{s-b} - \frac{1}{s-c}\right]\delta C.$$

(c) Find the minimum distance from the point (1, 2, 0) to the cone $z^2 = x^2 + y^2$.

4. Attempt any **FOUR** parts of the following : **(5×4=20)**

(a) Evaluate $\iiint_R (x - 2y + z)\,dz\,dy\,dx$, where R is the region determined by $0 \le x \le 1$, $0 \le y \le x^2$, $0 \le z \le x + y$.

(b) Find the area lying between the parabola $y = 4x - x^2$, and the line $y = x$.

(c) Evaluate by changing the order of integration :

$$\int_0^1 \int_{2y}^2 e^{x^2}\, dx\,dy.$$

(d) Define Gamma and Beta functions. Prove that :

$$\beta(\ell, m)\cdot\beta(\ell + m, n)\cdot\beta(\ell + m + n, p) = \frac{T(\ell)\,T(m)\,T(n)\,T(p)}{T(\ell + m + n + p)}.$$

(e) Express the following integral in terms of Beta function

$$\int_0^1 \frac{x^2}{\sqrt{1 - x^5}}\,dx.$$