Printed Pages—4 **EAS103**

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9601	Roll No.										

B. Tech.

(SEM. I) ODD SEMESTER THEORY EXAMINATION 2013-14

MATHEMATICS—I

Time : 3 Hours *Total Marks : 100*

SECTION—A

1. Attempt **all** parts of this question : **(10×2=20)**

(a) If $z = u^2 + v^2$ and $u = at^2$, $v = 2at$, then find $\frac{dz}{dt}$.

(b) If $y = x^2 e^x$, then find y_n.

(c) If $u = lx + my$, $v = mx - ly$, then find $\frac{\partial(x, y)}{\partial(u, v)}$.

(d) Find the stationary points of

$$f(x, y) = 5x^2 + 10y^2 + 12xy - 4x - 6y + 1.$$

(e) Find the rank of the following matrix by reducing into Echelon form :

$$\begin{bmatrix} 3 & 1 & 2 \\ 6 & 2 & 4 \\ 3 & 1 & 2 \end{bmatrix}.$$

(f) Find the eigen values of A^2 where $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 3 & 4 & 5 \end{bmatrix}$.

(g) Evaluate $\Gamma(3/4)\cdot\Gamma(1/4)$.

(h) Use Dirichlet's integral to evaluate $\iiint xyz\,dx\,dy\,dz$, throughout the volume bounded by $x=0, y=0, z=0$ and $x+y+z=1$.

(i) If $\vec{r}$ is the position vector of a point, then find $\text{Curl}\,\vec{r}$.

(j) Find the unit normal to the surface $x^2y + 2xz = 4$ at the point $(2, -2, 3)$.

SECTION—B

2. Attempt any **three** parts of the following : **(3×10=30)**

(a) If $y = e^{a\cos^{-1}x}$, then find y_n for $x = 0$.

(b) A rectangular box, open at the top, is to have a volume of 32 c.c. Find the dimensions of the box requiring least material for its construction.

(c) Find the characteristic roots of the matrix $A = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix}$. Verify Cayley-Hamilton theorem for this matrix and hence find A^{-1}.

(d) Change the order of integration in the following integral and hence evaluate it :

$$\int_0^1 \int_{y^2}^{2-y} xy\,dx\,dy.$$

(e) Evaluate $\iint_S (y^2z^2\hat{i} + z^2x^2\hat{j} + z^2y^2\hat{k})\cdot\hat{n}\,dS$, where S is the part of the sphere $x^2 + y^2 + z^2 = 1$ above the xy-plane and bounded by the xy-plane.

SECTION—C

Attempt any **two** parts from each question of this section. All questions are compulsory : **[(2×5)×5=50]**

3. (a) Verify Euler's theorem for $u = \dfrac{x(x^4 - y^4)}{x^4 + y^4}$.

(b) Find the Taylor's series expansion of $f(x, y) = x^2y + \sin y + e^x$ about the point $(1, \pi)$ upto second degree terms.

(c) Trace the curve $y = x(x^2 - 1)$.

4. (a) If $x = r\cos\theta, y = r\sin\theta$, then prove that $JJ' = 1$.

(b) Find the maximum percentage error in the time period of a simple pendulum due to possible errors upto 1% in l and 2.5% in g.

(c) If $u = y + z$, $v = x + 2z^2$, $w = x - 4yz - 2y^2$, then find $\dfrac{\partial(u, v, w)}{\partial(x, y, z)}$. Are u, v and w functionally related ? If so, find the relationship.

5. (a) Find the rank of the following matrix by reducing it into Normal form :

$$A = \begin{bmatrix} -1 & -3 & 3 & -1 \\ 1 & 1 & -1 & 0 \\ 2 & -5 & 2 & -3 \end{bmatrix}.$$

(b) Show that the following equations are consistent and solve them :

$$x + 2y - z = 3;$$
$$3x - y + 2z = 1;$$
$$2x - 2y + 3z = 2;$$
$$x - y + z = -1.$$

(c) Find the eigen vectors for the matrix $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$.

6. (a) Prove that :

$$\int_0^{\pi/2} \frac{dx}{\sqrt{\sin x}} \cdot \int_0^{\pi/2} \sqrt{\sin x}\, dx = \pi.$$

(b) Evaluate $\int_0^{2a} \int_0^{\sqrt{2ax - x^2}} (x^2 + y^2)\, dxdy$ by changing into polar coordinates.

(c) Determine the area bounded by the curves $xy = 2$, $4y = x^2$ and $y = 4$.

7. (a) Find the directional derivative of div(grad f) at the point (1, –2, 1) in the direction of the normal to the surface $xy^2z = 3x + z^2$, where $f = 2x^3y^2z^4$.

(b) Find the work done in moving a particle in the force field :

$$\vec{F} = 3x^2\hat{i} + (2xz - y)\hat{j} + z\hat{k}$$

along the curve $x^2 = 4y$ and $3x^3 = 8z$ from $x = 0$ to $x = 2$.

(c) Find the constants a, b, c so that :

$$\vec{F} = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + cy + 2z)\hat{k}$$

is irrotational.