Printed Pages: **189** **NAS-301**

(Following Paper ID and Roll No. to be filled in your Answer Book)

Paper ID :199103 Roll No. | | | | | | | | | | |

B.Tech.

(SEM. I) THEORY EXAMINATION, 2015-16

ENGINEERING MATHEMATICS-I

[Time:3 hours] **[Total Marks:100]**

Section-A

Q.1 Attempt all parts. All parts carry equal marks. Write answer of each part in shorts. (10×2=20)

(a) If $Y=e^{\sin^{-1}}x$, find the value of $(1-x^2)y_2-xy_1-a^2y$.

(b) If $V=(x^2+y^2+z^2)^{-1/2}$, then find $x\frac{\partial v}{\partial x}+y\frac{\partial v}{\partial y}+z\frac{\partial z}{\partial x}$.

(c) If f(x,y,z,w)=0, then find $\frac{\partial x}{\partial y}\times\frac{\partial y}{\partial z}\times\frac{\partial z}{\partial w}\times\frac{\partial w}{\partial x}$.

(d) If $pv^2=k$ and the relative errors in p and v are respetively 0.05 and 0.025, show that the error in k is 10%.

(e) Examine whether the vectors $x_1=[3,1,1]$, $x_2=[2,0,-1]$, $x_3=[4,2,1]$ are linearly independent.

(f) If $A=\begin{bmatrix}-1 & 0 & 0\\ 2 & -3 & 0\\ 1 & 4 & -2\end{bmatrix}$, find the eigen values of A^2.

(g) Evaluate $\int_0^1\int_0^1 \frac{dxdy}{\sqrt{1-x^2}\sqrt{1-y^2}}$

(h) Find the value of integral $\int_0^\infty e^{-ax}x^{n-1}dx$.

(i) Find the curl of $\vec{F}=xy\hat{i}+y^2\hat{j}+xz\hat{k}$ at (-2,4,1)

(j) State Stoke's theorem.

Section-B

Attempt any five Questions from this section:

(5x10=50)

Q.2. If $\cos^{-1}x=\log(y)^{1/m}$, then show $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(n^2+m^2)y_n=0$ and hence Calculate Y_n when $x=0$.

Q.3 If u,v,w are the roots of the equation $(\lambda-x)^3+(\lambda-y)^3+(\lambda-z)^3=0$ in λ find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$

Q.4 Using the Lagrange's method find the dimension of rectangular box of maximum capacity whose surface area is given when (a) box is open at the top (b) box is closed.

Q.5 Find the characteristic equation of the matrix

$A=\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}$ and verify Cayley Hamilton theorem.

Also evaluate $A^6-6A^5+9A^4-2A^3-12A^2+23A-9I$.

Q.6 Prove that $\iiint \frac{dx\,dy\,dz}{\sqrt{(1-x^2-y^2-z^2)}}=\frac{\pi^2}{8}$, the integral being extended to all positive values of the variables for which the expression is real.

Q.7 Verify the Green's theorem to evaluate the line integral $\int_C(2y^2dx+3xdy)$, where C is the boundary of the closed region bounded by $y=x$ and $y=x^2$.

Q.8 Determine the values 'a' and 'b' for which the following system of equation has.

$$x+y+z=6$$
$$x+2y+3z=10,$$
$$x+2y+az=b$$

(i) No solution

(ii) A unique solution

(iii) Infinite no of solutions.

Q.9 Find the mass of a solid $\left(\frac{x}{ab}\right)^p+\left(\frac{y}{b}\right)^q+\left(\frac{z}{c}\right)^r=1$, the density at any pòint being $p=kx^{l-1}y^{m-1}z^{n-1}$ where x, y, z are all positive.

Section-C

Attempt any two questions from this section: (2×15=30)

Q10. a) If $u=f(r)$ where $r^2=x^2+y^2$, show that

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=f''(r)+\frac{1}{r}f'(r).$$

b) Change the order of Integration in $I=\int_0^1\int_{x^2}^{2-x} xy\,dxdy$ and hence evalute.

c) Find the rank of the matrix by reducing to normal form. $\begin{pmatrix} 3 & 2 & -1 \\ 4 & 2 & 6 \\ 7 & 4 & 5 \end{pmatrix}$

Q.11 a) A fluid motion is given by $\bar{v}=(y+z)\,\hat{i}+(z+x)\hat{j}+(x+y)\,\hat{k}$. Show that the motion is irrotational and hence find the velocity potential.

b) If $x+y+z=u,\ y+z=uv,\ z=uvw$ then find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$.

c) Prove that, for every field $\bar{V}$; div curl $\bar{V}=0$.

Q.12 a) Evaluate $\iiint_R (x+y+z)dx\,dy\,dz$ where

$$R: 0\le x\le 1;\ 1\le y\le 2;\ 2\le z\le 3.$$

b) Trace the curve $y^2(2a-x)=x^3$.

c) Verify Euler's theorem for the function

$$Z = \frac{x^{\frac{1}{3}} + y^{\frac{1}{3}}}{x^{\frac{1}{2}} + y^{\frac{1}{2}}}.$$

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