Printed Pages: 03 **Sub Code: KAS103**

Paper Id: 19910 **Roll No.**

B.Tech.
(SEM-I) THEORY EXAMINATION 2018-19
MATHEMATICS-I

Time: 3 Hours ***Total Marks: 100***

Note: Attempt all Sections. If require any missing data; then choose suitably.

SECTION A

1. Attempt *all* questions.

Q no.	Question	Marks	CO
a.	Find the rank of the matrix $\begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix}$.	2	1
b.	Find the stationary point of $f(x,y) = x^3 + y^3 + 3axy, a > 0$	2	3
c.	If $x = r\cos\theta,\ y = r\sin\theta, z = z$ then find $\frac{\partial(r,\theta,z)}{\partial(x,y,z)}$	2	3
d.	Define del ∇ operator and gradient.	2	5
e.	If $\phi = 3x^2y - y^3z^2$, find grad ϕ at point (2, 0, -2).	2	
f.	Evaluate $\int_0^1 \int_0^{x^2} e^{\frac{y}{x}}\, dxdy$	2	4
g.	If the eigen values of matrix A are 1, 1, 1, then find the eigen values of $A^2 + 2A + 3I$.	2	1
h.	Define Rolle's Theorem	2	2
i.	If $u = x^3y^2\sin^{-1}(y/x)$, then find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$.	2	3
j.	In RI = E and possible error in E and I are 20% and 10 % respectively, then find the error in R.	2	3
k.	State the Taylor's Theorem for two variables.	2	3

SECTION B

2. Attempt any *three* of the following:

Q no.	Question	Marks	CO
a.	Using Cayley- Hamilton theorem find the inverse of the matrix A= $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$. Also express the polynomial B= $A^8 - 11A^7 - 4A^6 + A^5 + A^4 - 11A^3 - 3A^2 + 2A + I$ as a quadratic polynomial in A and hence find B.	10	1

		Marks	CO
b.	If $y = \sin(m \sin^{-1}x)$, prove that : $(1 - x^2) y_{n+2} - (2n + 1)x\, y_{n+1} - (n^2 - m^2)y_n = 0$ and find y_n at $x = 0$.	10	2
c.	If u, v, w are the roots of the equation $(x-a)^3 + (x-b)^3 + (x-c)^3 = 0$, then find $\frac{\partial(u,v,w)}{\partial(a,b,c)}$.	10	3
d.	Evaluate $\int_0^{\infty}\int_0^{\infty} e^{-(x^2+y^2)} dxdy$ by changing to polar coordinates. Hence show that $\int_0^{\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$.	10	4
e.	Verify the divergence theorem for $\bar{F} = (x^3 - yz)\hat{i} + (y^3 - zx)j + (z^3 - xy)\hat{k}$, taken over the cube bounded by planes x = 0, y = 0, z = 0, x = 1, y = 1, z = 1.	10	5

SECTION C

3. Attempt any *one* part of the following:

Q no.	Question	Marks	CO
a.	Find inverse employing elementary transformation $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$	10	1
b.	Reduce the matrix A to its normal form when $A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ -1 & -2 & 6 & -7 \end{bmatrix}$. Hence find the rank of A.	10	1

4. Attempt any *one* part of the following:

Q no.	Question	Marks	CO
a.	If $\sin^{-1} y = 2\log(x+1)$ show that $(x+1)^2 y_{n+2} + (2n+1)(x+1)y_{n+1} + (n^2+4)y_n = 0$	10	2
b.	Verify Lagrange's Mean value Theorem for the function $f(x) = x^3$ in $[-2,2]$	10	2

5. Attempt any *one* part of the following:

Q no.	Question	Marks	CO
a.	Find the maximum or minimum distance of the point (1, 2, -1) from the sphere $x^2 + y^2 + z^2 = 24$.	10	3
b.	If $u = \cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \frac{1}{2}\cot u = 0$	10	3

6. Attempt any *one* part of the following:

Q no.	Question	Marks	CO
a.	Change the order of integration and then evaluate: $\int_{0}^{2}\int_{\frac{x^2}{4}}^{3-x} xy\, dy\, dx$.	10	4
b.	Calculate the volume of the solid bounded by the surface x=0, y=0, x+y+z=1 & z=0.	10	4

7. Attempt any *one* part of the following:

Q no.	Question	Marks	CO
a.	Prove that $(y^2 - z^2 + 3yz - 2x)\hat{i} + (3xz + 2xy)\hat{j} + (3xy - 2xz + 2z)\hat{k}$ is both Solenoidal and Irrotational.	10	5
b.	Find the directional derivative of $\Phi = 5x^2y - 5y^2z + \frac{5}{2}z^2x$ at the point P(1, 1, 1) in the direction of the line $\frac{x-1}{2} = \frac{y-3}{-2} = \frac{z}{1}$.	10	5