

Paper Id: 199103Roll No:

B. TECH.
(SEM I) THEORY EXAMINATION 2019-20
MATHEMATICS-I

Time: 3 Hours

Total Marks: 100

Note: 1. Attempt all Sections. If require any missing data; then choose suitably.

SECTION A

1. Attempt all questions.

Q. No.	Question	Marks	CO
a.	Show that vectors (1, 6, 4), (0, 2, 3) and (0, 1, 2) are linearly independent.	2	1
b.	Define Lagrange's mean value theorem.	2	2
c.	If $u = x(1 - y)$, $v = xy$, find $\frac{\partial(u,v)}{\partial(x,y)}$.	2	3
d.	Show that vector $\vec{V} = (x + 3y)\hat{i} + (y - 3z)\hat{j} + (x - 2z)\hat{k}$ is solenoidal.	2	5
e.	Find the value of 'b' so that rank of $A = \begin{bmatrix} 2 & 4 & 2 \\ 3 & 1 & 2 \\ 1 & 0 & b \end{bmatrix}$ is 2.	2	1
f.	Evaluate $\int_0^2 \int_0^1 (x^2 + 3y^2) dy dx$.	2	4
g.	Find grad ϕ at the point (2, 1, 3) where $\phi = x^2 + yz$	2	5
h.	If $u = \cos^{-1} \left(\frac{x+y}{\sqrt{x}+\sqrt{y}} \right)$, then find the value of $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$.	2	3
i.	Find $\frac{du}{dt}$ if $u = x^3 + y^3$, $x = a \cos t$, $y = b \sin t$.	2	3
j.	Find the area lying between the parabola $y = 4x - x^2$ and above the line $y = x$.	2	4

SECTION B

2. Attempt any three of the following:

Q. No.	Question	Marks	CO
a.	Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 4 & 0 & 1 \\ 0 & 1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$ and hence find A^{-1} .	10	1
b.	If $y = e^{m \cos^{-1} x}$, prove that $(1 + x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + m^2)y_n = 0$. Hence find y_n when $x = 0$.	10	2
c.	If $u^3 + v^3 + w^3 = x + y + z$, $u^2 + v^2 + w^2 = x^3 + y^3 + z^3$ and $u + v + w = x^2 + y^2 + z^2$, then show that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = \frac{(x-y)(y-z)(z-x)}{(u-v)(v-w)(w-u)}$.	10	3
d.	Evaluate the integral by changing the order of integration: $I = \int_0^1 \int_{x^2}^{2-x} xy \, dy \, dx$.	10	4
e.	Verify Stoke's theorem for the vector field $\vec{F} = (x^2 - y^2)\hat{i} + 2xy\hat{j}$ integrated round the rectangle in the plane $z = 0$ and bounded by the lines $x = 0, y = 0, x = a, y = b$.	10	5

SECTION C

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3. Attempt any one part of the following:

Q. No.	Question	Marks	CO
a.	For what values of λ and μ the system of linear equations: $x + y + z = 6$ $x + 2y + 5z = 10$ $2x + 3y + \lambda z = \mu$ has (i) a unique solution (ii) no solution (iii) infinite solution Also find the solution for $\lambda = 2$ and $\mu = 8$.	10	1
b.	Find the rank of the matrix $A = \begin{bmatrix} 1 & 3 & 4 & 2 \\ 2 & -1 & 3 & 2 \\ 3 & -5 & 2 & 2 \\ 6 & -3 & 8 & 6 \end{bmatrix}$ by reducing it to normal form.	10	1

4. Attempt any one part of the following:

Q. No.	Question	Marks	CO
a.	Verify the Cauchy's mean value theorem for the function e^x and e^{-x} in the interval $[a, b]$. Also show that 'c' is the arithmetic mean between a and b.	10	2
b.	Trace the curve $r^2 = a^2 \cos 2\theta$.	10	2

5. Attempt any one part of the following:

Q. No.	Question	Marks	CO
a.	If $u = f(2x - 3y, 3y - 4z, 4z - 2x)$, prove that $\frac{1}{2} \frac{\partial u}{\partial x} + \frac{1}{3} \frac{\partial u}{\partial y} + \frac{1}{4} \frac{\partial u}{\partial z} = 0$.	10	3
b.	Find the volume of the largest rectangular parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.	10	3

6. Attempt any one part of the following:

Q. No.	Question	Marks	CO
a.	Evaluate $\iint (x + y)^2 dx dy$, where R is the parallelogram in the xy-plane with vertices $(1, 0), (3, 1), (2, 2), (0, 1)$ using the transformation $u = x + y, v = x - 2y$.	10	4
b.	Find the volume of the region bounded by the surface $y = x^2, x = y^2$ and the planes $z = 0, z = 3$.	10	4

7. Attempt any one part of the following:

Q. No.	Question	Marks	CO
a.	Verify the divergence theorem for $\vec{F} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$ taken over the rectangular parallelepiped $0 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq z \leq 1$.	10	5
b.	Find the directional derivative of $\phi(x, y, z) = x^2yz + 4xz^2$ at $(1, -2, 1)$ in the direction of $2\hat{i} - \hat{j} - 2\hat{k}$. Find also the greatest rate of increase of ϕ .	10	5