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BTECH
(SEM II) THEORY EXAMINATION 2021-22
ENGG MATHEMATICS-II

Time: 3 Hours   *Total Marks: 100*

Notes:
- *Attempt all Sections and assume any missing data.*
- *Appropriate marks are allotted to each question, answer accordingly.*

SECTION-A	Attempt **All** of the following Questions in brief	Marks(10X2=20)
Q1(a)	Find the particular integral of $(D^2 – 4)y = x^2$, where $D=d/dx$	
Q1(b)	Find the order and degree of the following differential equation $$\frac{d^2 y}{dx^2}+\sqrt{1+\left(\frac{dy}{dx}\right)^2}=0$$	
Q1(c)	For a Legendre polynomial prove that $P_n(1)=1$ and $P_n(-1)=(-1)^n$.	
Q1(d)	Write the Dirichlet's conditions for Fourier series.	
Q1(e)	Prove that $L[e^{at} f(t)] = F(s-a)$.	
Q1(f)	Write the two dimensional wave equations.	
Q1(g)	Find the inverse Laplace transform of $\dfrac{s}{s^2+9}$.	
Q1(h)	Find the value of the Fourier coefficient a_0 for the function $$f(x)=\begin{cases} 0, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$$	
Q1(i)	Classify the following partial differential equation along the line $y = x$: $$y\,u_{xx} + (x+y)\,u_{xy} + x\,u_{yy} = 0$$	
Q1(j)	Show that: $J_{1/2}(x) = J_{-1/2}(x) \cot x$.	

SECTION-B	Attempt **ANY THREE** of the following Questions	Marks(3X10=30)
Q2(a)	Solve the following simultaneous differential equations $$\frac{d^2 x}{dt^2} + y = \sin t, \quad \frac{d^2 y}{dt^2} + x = \cos t,$$	
Q2(b)	Obtain the Fourier series to represent $f(x) = \frac{1}{4}(\pi - x)^2$ in the interval $0 \le x \le 2\pi$. Hence deduce that $\dfrac{1}{1^2} + \dfrac{1}{2^2} + \dfrac{1}{3^2} + \dfrac{1}{4^2} + ... = \dfrac{\pi^2}{6}$	
Q2(c)	Find the series solution for the differential equation $(1 - x^2)y'' - 2xy' + 20y = 0$.	
Q2(d)	State Convolution theorem for the inverse Laplace transforms and hence evaluate $$L^{-1}\left\{\frac{1}{(s^2+a^2)^2}\right\}.$$	
Q2(e)	A tightly stretched string with fixed end x=0 and x=l is initially in a position given by $y = a \sin \dfrac{\pi x}{l}$. If it is released from rest this position, find the displacement $y(x,t)$.	

SECTION-C	Attempt **ANY ONE** following Question	Marks (1X10=10)
Q3(a)	Solve by the method of variation of parameters: $\dfrac{d^2 y}{dx^2} + 4y = \sec 2x$	

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Q3(b)	Solve $x^2 \dfrac{d^3 y}{dx^3} + x \dfrac{d^2 y}{dx^2} + \dfrac{dy}{dx} = x^2 \log x$.

SECTION-C	Attempt **ANY ONE** following Question	Marks **(1X10=10)**
Q4(a)	Prove that $(n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x)$, where $P_n(x)$ is the Legendre's function.	
Q4(b)	Find the series solution of the differential equation $2x^2 \dfrac{d^2 y}{dx^2} + x \dfrac{dy}{dx} - (x+1)y = 0$.	

SECTION-C	Attempt **ANY ONE** following Question	Marks **(1X10=10)**
Q5(a)	Using Laplace transform, find the solution of the initial value problem $\dfrac{d^2 x}{dt^2} + 2\dfrac{dx}{dt} + 5x = e^{-t}\sin t$, where $x(0)$=0 and $x'(0) = 1$.	
Q5(b)	Express $f(t) = \begin{cases} 2t, & 0 < t \le \pi \\ 8, & t > 5 \end{cases}$ in terms of unit step function and hence obtain Laplace transform.	

SECTION-C	Attempt **ANY ONE** following Question	Marks **(1X10=10)**
Q6(a)	Find the Fourier series expansion of the periodic function $f(x) = x\cos x$, $-\pi < x < \pi$.	
Q6(b)	Solve the partial differential equation $(D^2 - D'^2 - 3D + 3D')z = xy + e^{x+2y}$.	

SECTION-C	Attempt **ANY ONE** following Question	Marks **(1X10=10)**
Q7(a)	Solve the P.D.E. by separation of variables method, $u_{xx} = u_y + 2u,\ u(0, y) = 0,\ u'(0, y) = 1 + e^{-3y}$.	
Q7(b)	Solve the Laplace equation $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0$ in a rectangle in the xy-plane with $u(x,0) = 0,\ u(x,b) = 0,\ u(0, y) = 0$ and $u(a, y) = f(y)$ parallel to y-axis.	