

Roll No:

| | | | | | | | | | | | |
|--|--|--|--|--|--|--|--|--|--|--|--|
| | | | | | | | | | | | |
|--|--|--|--|--|--|--|--|--|--|--|--|

BTECH
(SEM II) THEORY EXAMINATION 2021-22
MATHEMATICS II

Time: 3 Hours**Total Marks: 100****Note:** Attempt all Sections. If you require any missing data, then choose suitably.**SECTION A****1. Attempt all questions in brief. 2*10 = 20**

| Q. No. | Questions | CO |
|--------|---|----|
| (a) | Solve $((D + 1)^3 y = 2e^{-x}$ | 1 |
| (b) | What are the roots of the indicial equation for the power series solution of the differential equation $2x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + (x^2 - 3)y = 0$ | 1 |
| (c) | Find the volume of hemisphere. | 2 |
| (d) | Examine the convergence of improper integral $\int_2^{\infty} \frac{1}{(x \log x)} dx$ | 2 |
| (e) | If $f(x) = 1, 0 < x < \pi$ is expanded in half range sine series then find the value of b_n . | 3 |
| (f) | Discuss the convergence of sequence $(1, 2^1, 2^2, 2^3, 2^4 \dots \dots)$. | 3 |
| (g) | Define Harmonic function. | 4 |
| (h) | Find the points of invariant of the transformation $w = \frac{2z+3}{z+2}$. | 4 |
| (i) | State Cauchy integral theorem. | 5 |
| (j) | Discuss the singularity of $\sin\left(\frac{1}{z-a}\right)$. | 5 |

SECTION B**2. Attempt any three of the following: 10*3 = 30**

| Q. No | Questions | CO |
|-------|---|----|
| (a) | Solve $x \frac{d^2 y}{dx^2} + (4x^2 - 1) \frac{dy}{dx} + 4x^3 y = 2x^3$ | 1 |
| (b) | Show that $\Gamma(m) \cdot \Gamma\left(m + \frac{1}{2}\right) = \frac{\sqrt{\pi}}{(2)^{2m-1}} \Gamma(2m)$ where m is positive. | 2 |
| (c) | Obtain Fourier series for $f(x) = \begin{cases} \pi x, & 0 \leq x \leq 1 \\ \pi(2-x), & 1 \leq x \leq 2 \end{cases}$ | 3 |
| (d) | Examine the nature of the function $f(z) = \begin{cases} \frac{x^2 y^5 (x+iy)}{x^4 + y^{10}}, & z \neq 0 \\ 0, & z = 0 \end{cases}$ in the region including the origin. | 4 |
| (e) | Evaluate $\frac{1}{2\pi i} \oint_C \frac{z^2 - z + 1}{z - 2} dz$, where $C \equiv z - 1 = \frac{1}{2}$ | 5 |

Roll No:

| | | | | | | | | | | | | | | |
|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|
| | | | | | | | | | | | | | | |
|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|

BTECH
(SEM II) THEORY EXAMINATION 2021-22
MATHEMATICS II

SECTION C

3. Attempt any *one* part of the following: 10*1 = 10

| Q. No | Questions | CO |
|-------|---|----|
| (a) | Solve by change of independent variable method $(1+x)^2 \frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 4\cos \log (1+x)$ | 1 |
| (b) | Solve the equations $t\frac{dy}{dt} + x = 0$ and $t\frac{dx}{dt} + y = 0$ given $x(0)=1$ and $y(-1) = 0$ | 1 |

4. Attempt any *one* part of the following: 10 *1 = 10

| | | |
|-----|--|---|
| (a) | Analyze the volume contained in the solid region in the 1 st octant of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ by applying Dirichlet's Integral | 2 |
| (b) | Establish the relation between Beta and Gamma function. | 2 |

5. Attempt any *one* part of the following: 10*1 = 10

| | | |
|-----|---|---|
| (a) | Find half range Fourier sine series for $f(x) = \begin{cases} x, & 0 < x < \pi/2 \\ \pi - x, & \pi/2 < x < \pi \end{cases}$ | 3 |
| (b) | Examine the series for convergence or divergence $1 + \frac{x}{2} + \frac{2!}{3^2}x^2 + \frac{3!}{4^3}x^3 + \dots$ | 3 |

6. Attempt any *one* part of the following: 10*1 = 10

| | | |
|-----|---|---|
| (a) | Define an analytic function. If $f(z) = u + iv$ is an analytic function find $f(z)$ in terms of z if $u - v = e^x(\cos y - \sin y)$. | 4 |
| (b) | Find the image of the circle $ z - 1 = 1$ in the complex plane under the mapping $w = 1/z$. | 4 |

7. Attempt any *one* part of the following: 10*1 = 10

| | | |
|-----|--|---|
| (a) | Find Laurent series expansion of $\frac{1-\cos z}{z^3}$ about the point $z = 0$ is | 5 |
| (b) | Find residue at each pole of the function $\frac{4+3z}{(z-2)(z-3)}$ and hence using Cauchy residue theorem evaluate integral $\oint_C \frac{4+3z}{(z-2)(z-3)} dz$, where $C: z = 1$ | 5 |